Parallel Polarization

Incident wave:

$$\tilde{E}^i_{\parallel} = \hat{y}_i E^i_{\parallel} e^{-jk_1 x_i} = (\hat{x}\cos\theta_i - \hat{z}\sin\theta_i) E^i_{\parallel 0} e^{-jk_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\tilde{H}^i_{\parallel} = \hat{y}\frac{E^i_{\parallel 0}}{\eta_1} e^{-jk_1 x_i} = \hat{y}\frac{E^i_{\parallel 0}}{\eta_1} e^{-jk_1(x\sin\theta_i + z\cos\theta_i)}$$

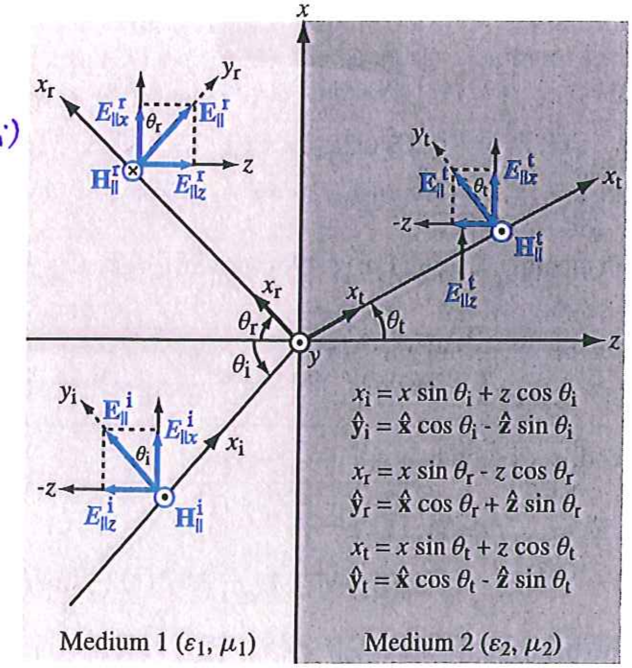


Figure 8-16: Parallel polarized plane wave incident at an angle θ_i upon a planar boundary.

Reflected wave:

$$\tilde{E}^r_{\parallel} = \hat{y}_r E^r_{\parallel 0} e^{-jk_1 x_r}$$

$$= (\hat{x}\cos\theta_r + \hat{z}\sin\theta_r) E^r_{\parallel 0} e^{-jk_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\tilde{H}^r_{\parallel} = -\hat{y}\frac{E^r_{\parallel 0}}{\eta_1} e^{-jk_1 x_r}$$

$$= -\hat{y}\frac{E^r_{\parallel 0}}{\eta_1} e^{-jk_1(x\sin\theta_r - z\cos\theta_r)}$$

Transmitted wave:

$$\tilde{E}^t_{\parallel} = \hat{y}_t E^t_{\parallel 0} e^{-jk_2 x_t} = (\hat{x}\cos\theta_t - \hat{z}\sin\theta_t) E^t_{\parallel 0} e^{-jk_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\tilde{H}^t_{\parallel} = \hat{y}\frac{E^t_{\parallel 0}}{\eta_2} e^{-jk_2 x_2} = -\hat{y}\frac{E^t_{\parallel 0}}{\eta_2} e^{-jk_2(x\sin\theta_t - z\cos\theta_t)}$$

Applying boundary condition at $z=0$: tangential components of E and H are equal.

$$\tilde{E}^i_{\parallel} + \tilde{E}^r_{\parallel}\Big|_{\text{tang. at } z=0} = \tilde{E}^t_{\parallel}\Big|_{\text{tang. at } z=0} \Rightarrow \tilde{E}^i_{\parallel x} + \tilde{E}^r_{\parallel x}\Big|_{z=0} = \tilde{E}^t_{\parallel x}\Big|_{z=0}$$

and same for $\tilde{H}$ but in y direction.

$$\Rightarrow \begin{cases} \cos\theta_i E^i_{\parallel 0} e^{-jk_1 x\sin\theta_i} + \cos\theta_r E^r_{\parallel 0} e^{-jk_1 x\sin\theta_r} = \cos\theta_t E^t_{\parallel 0} e^{-jk_2 x\sin\theta_t} \\ \dfrac{E^i_{\parallel 0}}{\eta_1} e^{-jk_1 x\sin\theta_i} - \dfrac{E^r_{\parallel 0}}{\eta_1} e^{-jk_1 x\sin\theta_r} = \dfrac{E^t_{\parallel 0}}{\eta_2} e^{-jk_2 x\sin\theta_t} \end{cases}$$

Solving the two equation we will get again the Snell's laws:

$$\begin{cases} \theta_i = \theta_r \\ k_1 \sin\theta_i = k_2 \sin\theta_t \end{cases}$$

as well as Fresnel reflection and transmission coefficient for parallel polarization:

$$\Gamma_{\parallel} = \frac{E^r_{\parallel 0}}{E^i_{\parallel 0}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\tau_{\parallel} = \frac{E^t_{\parallel 0}}{E^i_{\parallel 0}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

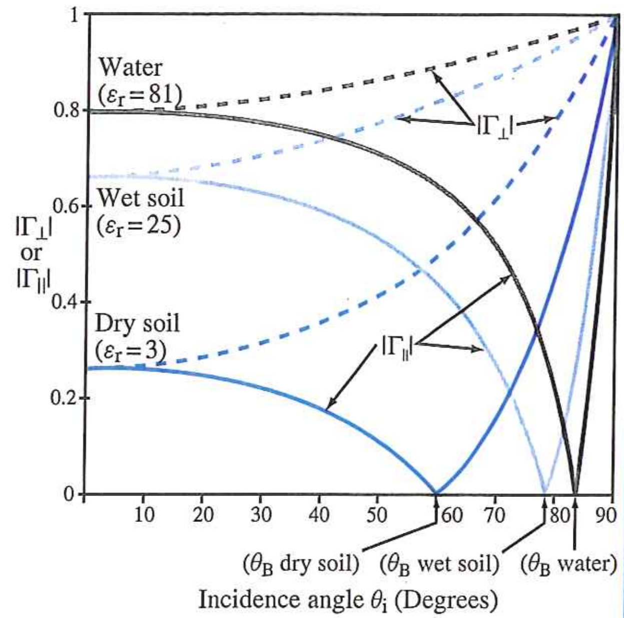


Figure 8-17: Plots for $|\Gamma_\perp|$ and $|\Gamma_\parallel|$ as a function of θ_i for a dry soil surface, a wet-soil surface, and a water surface. For each surface, $|\Gamma_\parallel| = 0$ at the Brewster angle.

$$\Rightarrow \quad \tau_{\parallel} = (1 + \Gamma_{\parallel}) \frac{\cos\theta_i}{\cos\theta_t}$$

As in perpendicular polarization, here also if medium 2 is a conductor $\eta_2 = 0$ and $\Gamma_{\parallel} = -1$ which means all the wave is reflected and $\tau_{\parallel} = 0$. We can also write for nonmagnetic materials ($\mu_1 = \mu_2 = \mu_0$):

$$\Gamma_{\parallel} = \frac{-\frac{\varepsilon_2}{\varepsilon_1}\cos\theta_i + \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}{\frac{\varepsilon_2}{\varepsilon_1}\cos\theta_i + \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}} \quad \text{for } \mu_1 = \mu_2$$

Moreover, we also observe that $\Gamma_{\parallel}$ depends on the angles θ_i and θ_t in a way that it can become zero for a certain angle when $\eta_2 \cos\theta_t - \eta_1 \cos\theta_i = 0$. It is also seen that:

1) $\theta_i = 0 \Rightarrow \Gamma_\perp = \Gamma_\parallel$ normal incident

2) $\theta_i = 90 \Rightarrow |\Gamma_\perp| = |\Gamma_\parallel| = 1$ grazing angle

3) $\theta_i = \theta_B \Rightarrow \Gamma_\parallel = 0$ Brewster angle → At the Brewster angle, the parallel-polarized component of the incident wave is totally transmitted into medium 2.

Brewster Angle, θ_B

θ_B is defined as the incident angle θ_i at which $\Gamma = 0$.

Picture for the example next page:

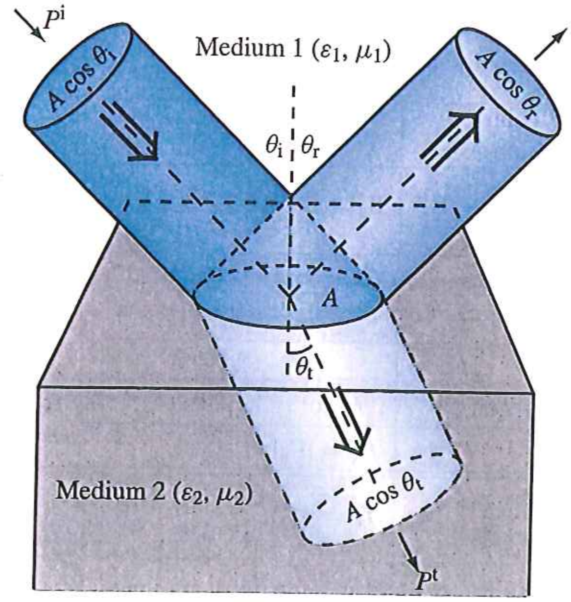


Figure 8-18: Reflection and transmission of an incident circular beam illuminating a spot of size A on the interface.

Perpendicular Polarization:

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = 0 \Rightarrow$$

$$\eta_2 \cos\theta_i = \eta_1 \cos\theta_t \rightarrow \eta_2^2 \cos^2\theta_i = \eta_1^2 (1 - \sin^2\theta_t)$$

$$\sqrt{\mu_2\varepsilon_2}\, \sin\theta_t = \sqrt{\mu_1\varepsilon_1}\, \sin\theta_i \rightarrow \sin\theta_t = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\, \sin\theta_i$$

$$\cos^2\theta_i = \left(\frac{\eta_1}{\eta_2}\right)^2 \left(1 - \frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2} \sin^2\theta_i\right)$$

$$1 - \sin^2\theta_i = \frac{\mu_1\varepsilon_2}{\varepsilon_1\mu_2}\left(1 - \frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}\sin^2\theta_i\right) \rightarrow \sin\theta_i = \sin\theta_{B\perp} = \sqrt{\frac{1 - \mu_1\varepsilon_2/\mu_2\varepsilon_1}{1 - \left(\frac{\mu_1}{\mu_2}\right)^2}}$$

For a non-magnetic material $\mu_1 = \mu_2$ and $\sin\theta_{B\perp} = \infty$, which means $\theta_{B\perp}$ does not exist. So for non-magnetic materials there is no θ_B for perpendicular polarization.

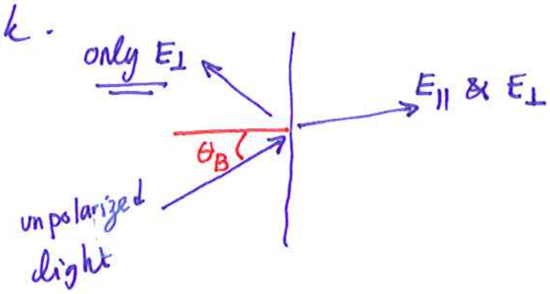
Parallel Polarization:

$$\Gamma_\parallel = 0 \Rightarrow \eta_2 \cos\theta_t = \eta_1 \cos\theta_i \Rightarrow \sin\theta_{B\parallel} = \sqrt{\frac{1 - (\varepsilon_1\mu_2/\varepsilon_2\mu_1)}{1 - (\varepsilon_1/\varepsilon_2)^2}}$$

For non-magnetic materials $\mu_1 = \mu_2 \Rightarrow$

$$\boxed{\theta_{B\parallel} = \sin^{-1}\frac{1}{\sqrt{1 + \frac{\varepsilon_1}{\varepsilon_2}}} = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \tan^{-1}\frac{n_2}{n_1}}$$

θ_B is also called polarizing angle as if a wave composed of both $\parallel$ and $\perp$ polarization is incident at a surface at θ_B, only the $\parallel$ polarization is transmitted through and only the perpendicular polarization is reflected back.



Example

A 5-W beam of light with circular cross section is incident in air upon a plane boundary of a dielectric medium with an index of refraction of 5. If the angle of incident is 60° and the incident wave is parallel polarized, determine the transmission angle and the powers contained in the reflected and transmitted beams.

$\theta_i = 60^\circ$

$n_2 = 5$

$n_1 = 1$

$$n_1 \sin\theta_i = n_2 \sin\theta_t \rightarrow \sin\theta_t = \frac{n_1}{n_2}\sin\theta_i = \frac{1}{5}\sin 60 = 0.17 \rightarrow \theta_t = 10^\circ$$

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{n_2^2}{n_1^2} = 5^2 = 25 \rightarrow \Gamma_{\parallel} = \frac{-\frac{\varepsilon_2}{\varepsilon_1}\cos\theta_i + \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}{\frac{\varepsilon_2}{\varepsilon_1}\cos\theta_i + \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}$$

$$= \frac{-25\cos 60 + \sqrt{25 - \sin^2 60}}{25\cos 60 + \sqrt{25 - \sin^2 60}} = -0.435$$

Reflected power:

$$P_{\parallel}^r = P_{\parallel}^i |\Gamma_{\parallel}|^2 = 5(0.435)^2 = 0.95\,W$$

Transmitted power:

$$P_{\parallel}^t = P_{\parallel}^i - P_{\parallel}^r = 5 - 0.95 = 4.05\,W$$